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Human observers are biased in judging the angular approach of a projectile

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Abstract

How do we decide whether an object approaching us will hit us? The optic array provides information sufficient for us to determine the approaching trajectory of a projectile. However, when using binocular information, observers report that trajectories near the mid-sagittal plane are wider than they actually are (J. Exp. Psych, 29 (2003) 869). Here we extend this work to consider stimuli containing additional depth cues. We measure observers' estimates of trajectory direction first for computer rendered, stereoscopically presented, rich-cue objects, and then for real objects moving in the world. We find that, under both rich cue conditions and with real moving objects, observers show positive bias, overestimating the angle of approach when movement is near the mid-sagittal plane. The findings question whether the visual system, using both binocular and monocular cues to depth, can make explicit estimates of the 3-D location and movement of objects in depth.

Keywords: Looming; Disparity; Motion-in-depth; Stereopsis

1. Introduction

One key function of the visual system is to provide information about objects moving in the environment that we might want to intercept or avoid. Object interception can be thought of as involving two related problems: (i) estimate the direction in which an object is moving and (ii) calculate the time at which it will arrive at a given point. Two sources of visual information that are potentially useful for estimating these parameters are the changing binocular horizontal disparity of an object (disparity cue), and the changing retinal size of an object (looming cue).

The role of looming and disparity cues in judgments of when an approaching object will hit us (time-to-contact) has been studied extensively (e.g. Heuer, 1993; Lee, 1976; Lee & Reddish, 1981; Lee, Young, Reddish, Lough, & Clayton, 1983; Regan & Hamstra, 1993; Todd, 1981). There is good evidence to suggest observers can make accurate estimates of time-to-contact by combining these sources of information according to the reliability of individual cues (Gray & Regan, 1998) or by giving highest weight to cues specifying a more imminent contact (Rushton & Wann, 1999).

The subject of this paper is the visual system's ability to make judgments of the motion trajectory of objects moving in the environment rather than time-to-contact judgments. The use of both looming (Regan & Kaushal, 1994) and horizontal disparity (Beverley & Regan, 1973, 1975; Portfors-Yeomans & Regan, 1996; Regan, Beverley, & Cynader, 1979) cues to the perception of angular approach of moving objects has previously been considered. A motivation for such studies is the apparently remarkable ability of people (especially expert sportsmen such as cricket players—Regan, 1992) to intercept rapidly moving projectiles. To understand the foundation of these abilities, the use of specific visual cues to the direction of motion in depth has been analysed in the laboratory. For instance, Portfors-Yeomans and

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Regan (1996) showed observers a reference motion-in-depth trajectory of a small object, presented in stereoscopic depth, followed by a test trajectory and they were asked to discriminate whether the test trajectory was wider of the head than the reference. They obtained discrimination thresholds of 0.4°–0.8° for motion trajectories near the median plane of the head. Beverley and Regan (1975) found even greater sensitivity to differences in motion-in-depth trajectories (ca. 0.2° for trajectories near the mid-sagittal plane). Regan and Kaushal (1994) used a similar method to evaluate the sensitivity of the visual system to motion-in-depth trajectories on the basis of the looming cue. They reported that just-noticeable-difference thresholds between sequentially presented trajectories could be less than 0.1°. These studies suggest that the human visual system can be remarkably precise in distinguishing between objects moving along different angular trajectories. However, measuring precision does not tell us about the bias, or accuracy, of observers.

One study has attempted to measure the accuracy of 3-D motion perception using horizontal binocular disparity to specify 3-D motion trajectories (Harris & Dean, 2003). Rather than using a relative task, where observers made a judgment of one trajectory with respect to another, they used an absolute task where the angular approach of a single object was judged with respect to the observer's own body. The results were quite surprising: observers dramatically overestimated the angle of approach in all cases except when the target approached the observer along the mid-sagittal plane. Although observers were not accurate, their reports of angular trajectory were precise (standard errors around 1°). Harris and Dean (2003) employed four different response measures and found poor accuracy in all cases. These results demonstrate that one cannot extrapolate from knowledge about the precision of behaviour and perception to absolute performance (i.e. biases) in behaviour and perception. Although observers may be very good at discriminating which of a pair of stimuli is, for instance, the larger, wider or brighter, this does not mean that the human visual system has an unbiased estimator of that source of information. To measure perceptual bias, judgments need to be made with respect to the observer's body or some other absolute reference.

The aim of this paper is to extend Harris and Dean's findings using richer visual stimuli to address whether the poor accuracy they found can be improved upon when stimuli contain more information about 3-D motion than relative horizontal binocular disparity alone. We do this by providing additional information in two principal ways. First, we used computer-rendered spheres of different sizes to investigate the role of looming cues when perceiving motion-in-depth. Harris and Dean (2003) employed small visual targets (8.3 arcmin) in which looming cues were too small to be useful (Gray & Regan, 1998). Second, we examined performance when observers view a target that moves in the real-world, thus avoiding the potential cue conflicts caused by the stereoscopic presentation of a stimulus on a computer monitor. Such conflicts are potentially important because they could lead to the motion excursion of the rendered object being underestimated. For instance, accommodative and blur cues would not change when observers view a computer-rendered trajectory, thus providing information that the rendered object was not approaching. Also, any small movements of the observer's head could provide motion parallax information that would specify that the object was at the constant distance of the screen. Combining these discrepant estimates of the object's distance could lead to an underestimation of the changing distance of the target and thus an overestimation of the presented trajectory (Eq. (A.1)).

Finally, we investigated whether information provided by tracking the moving object with the eyes could lead to reduced bias when judging angular trajectories. To do this we required observers in both experiments to either (1) perform the standard laboratory technique of keeping the eyes fixated on a point in the scene; or (2) instructed observers to follow the target with their eyes.

To summarise, in this paper we present two experiments. In the first, we studied trajectory perception using computer rendered, stereoscopically presented, textured balls moving towards the observers in depth. To determine whether looming cues were of critical importance in interpreting motion-in-depth trajectories we used several different sized objects. In the second experiment we studied the perception of motion-in-depth trajectories with real objects moving towards the observer to eliminate the possibility of an artefact due to the mode of presentation. To anticipate the results, we find overestimates of angular trajectory in all cases when the motion is within ±16° from the mid-sagittal plane.

Before presenting the experimental reports we briefly consider the mathematical relationships between visual information and the angular trajectory of a moving object.

2. Mathematical analysis

Here we consider the means by which observers could use the visual information provided by the movement of an object to calculate its real-world trajectory. Most studies of 3-D motion perception have considered looming and binocular disparity as separate sources of visual information. For instance, Regan (1993) and Harris and Dean (2003) both provided a mathematical analysis describing how disparity information might be used in 3-D trajectory perception. However, their analyses were specific to small target points, moving near the mid-sagittal plane. This is inappropriate for our purposes as we used larger objects that were sometimes far from the mid-sagittal plane. Regan and Kaushal (1994) did consider the use of changing retinal size information. However, their analysis was for a purely monocular situation.

Here we provide equations for the joint use of looming and binocular cues for the case of a spherical object approaching an observer. We formulate our analysis of binocular information in terms of the horizontal size ratio (HSR) of the images in the left and right eyes. The HSR is defined here as the horizontal angular size of an object in the right eye, divided by the horizontal angular size in the left eye.

Our use of this descriptor is slightly different from that used elsewhere (e.g. Banks & Backus, 1998; Rogers & Bradshaw, 1993) as we are interested in non-planar objects. The fact that our stimuli are non-planar means that there are monocular, non-corresponding "Da Vinci" regions on the left and right sides of the object's retinal image in the two eyes (i.e. the right eye sees more of the right side of the object than the left eye and vice versa—see Fig. 1B). These regions are known to provide the visual system with useful information (e.g. Nakayama & Shimojo, 1990). As the horizontal size ratio we formulate includes these monocular regions, the visual system has to calculate size ratios for non-corresponding points on the object. However, the visual system routinely deals with non-corresponding image properties.

The analysis we present is intended to demonstrate a possible way in which the visual system could use binocular information; we do not attempt to distinguish between different sources of binocular information (disparity information or size information), nor are we asserting that the visual system actually implements the equations we provide, or that the process of estimating a trajectory must rely solely on this specific formulation. Our aim is to demonstrate an exemplar for the way in which the available information could be used.

Consider a situation in which a spherical object approaches an observer at angle β to the mid-sagittal plane (Fig. 1). The observer could in principle use information about the changing azimuth (visual direction) of the ball (ϕ) coupled with information about the changing retinal size as the object moves. There are (at least) two ways in which retinal size information could be used. First, by taking the average of the retinal size of the object in each eye, the visual system could monitor how the object's mean retinal size changes over time to estimate the object's translation in depth (commonly referred to as looming). Angular trajectory can be estimated using the following equation (see Appendix A for derivation):

$$\tan\beta \approx \frac{\theta_0 \sin\phi}{\theta - \theta_0 \cos\phi} \quad (1)$$

where θ_0 is the initial angular size of half the object, θ the angular size of half the object after movement, and ϕ is the cyclopean azimuth (azimuth measured from a point halfway between the eyes: the cyclopean point). This is strictly a monocular formulation for obtaining β. The equation uses the small angle approximation,

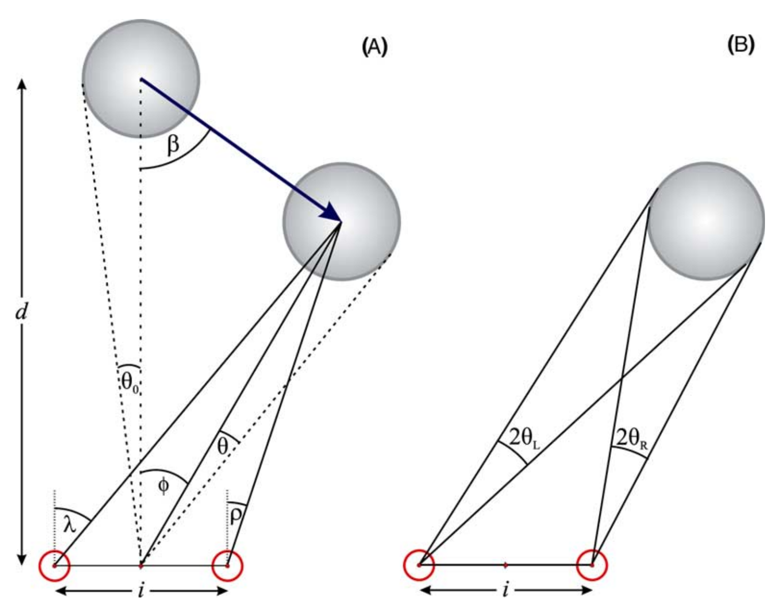

Fig. 1. (A) An observer views a sphere, initially located in the mid-sagittal plane at distance d, that moves at angle β. The nodal points of the eyes lie on the interocular axis separated by distance i. The cyclopean point is defined as the mid-point between the eyes on the interocular axis. The initial angular size of the object at the cyclopean point is $2\theta_0$, whilst after the movement it is 2θ. The azimuth of the centre of the object is λ in the left eye, ρ in the right eye and ϕ at the cyclopean point. (B) An illustration of size disparities. Horizontal angular size ratio is the angular size in the right eye $2\theta_R$ divided by angular size in the left eye $2\theta_L$.

requiring that the distance to the object is considerably larger than the object's size.

The second way that size information could be used is by taking account of the difference between right and left eyes' views of the stimulus. As the object moves out of the mid-sagittal plane the distance of the object from each eye will no longer be equal, producing a larger retinal image in one eye (retinal size disparity—see Fig. 1B). The observer could use the ratio of the image sizes in each eye (cf. Backus, Banks, van Ee, Crowell, & Crowell, 1999; Banks & Backus, 1998; Rogers & Bradshaw, 1993) to provide an estimate of the object's translation. Using this horizontal size ratio (retinal size in the right eye divided by retinal size in the left eye) the observer could calculate the angular trajectory using the following equation (see Appendix A for derivation):

$$\tan\beta \approx \frac{i\tan\phi(\mathrm{HSR}\sin\lambda + \sin\rho)}{2d\tan\phi(\mathrm{HSR}\sin\lambda - \sin\rho) - i(\mathrm{HSR}\sin\lambda + \sin\rho)} \quad (2)$$

where λ is the azimuth with respect to the left eye after object translation, ρ the azimuth with respect to the right eye after object translation, i is the interocular spacing and d the initial distance of the object. This equation uses the small angle approximation, requiring that the distance to the object is considerably larger than the object's size.

We should note here that we are not asserting that the human brain necessarily uses the HSR information, as expressed in Eq. (2), but rather that it could in principle. This is an example of how binocular information could be used for an extended object. Note also that for an object that is small enough to be approximated by a point, the HSR tends to 1. In this case, Eq. (1) reduces to the simple equation suggested by Harris and Dean (2003), which provides an expression for angle beta in terms of the relative horizontal binocular disparity (δ) between a target point, and a reference point in the scene:

$$\tan\beta \approx \frac{i\phi}{d\delta} \quad (3)$$

One notable difference between Eqs. (1) and (2) is that Eq. (1) requires only information that is specified on the retina, whereas Eq. (2) additionally requires knowledge of the viewing distance and the interocular separation. This difference is potentially important as it shows that monocular information (from looming) could in principle, allow the calculation of 3-D trajectory based on uncalibrated retinal information. If the visual system implemented such a scheme, then provided that retinal signals are measured correctly, trajectory estimates calculated from looming information should be unbiased. The use of horizontal binocular information (here expressed as HSRs), requires knowledge of interocular separation and viewing distance (or their ratio). Thus, even if retinal signals were measured correctly, errors in the estimation of viewing distance or interocular spacing could lead to biases in the observer's estimate of trajectory. We return to this point in the discussion.

3. Experiment 1: Rich-cue, computer-simulated objects

In this experiment we investigated the perception of 3-D angular trajectory using an absolute task for objects moving towards an observer when several sources of visual information were available. We investigated the influence of looming information in addition to binocular disparity information.

3.1. Methods

3.1.1. Apparatus

Stereoscopic presentation was achieved using a haploscope in which the two eyes viewed separate 21-in. CRT displays (Sony GDM-F520) through front-silvered mirrors. Viewing distance to the CRTs was 48 cm, and the haploscope was configured to promote the correct vergence angle. Stimuli were generated and presented on a Windows PC containing a nVidia GMX 420 graphics card. The graphics card displayed 1600×1200 pixels at a rate of 60 Hz. The size of individual pixels was 1.75 arcmin. Images were drawn on the CRT using only the electron gun configured to excite red phosphor. A red filter was placed in front of each CRT to remove residual light from the 'black' screen. Each CRT's brightness and contrast controls were adjusted to produce a low luminance output (max 4.05 cd/m^2). The monitor's controls were adjusted iteratively so that photometric measurements of light intensity produced by the CRTs were very similar on the two monitors. Standard techniques for the linearization of the video palette were employed. Head stabilization was achieved using a chin rest.

Measures of the perceived angle of approach were collected using a purpose built pointing device that consisted of a metal beam pivoted in the centre of a 25 cm circular casing. The pivot of the pointer was located at the virtual starting point of the motion trajectory presented to the observers. The angular rotation of the pointer was measured using a potentiometer attached to pointer at the pivot. Angular rotation was recorded by an experimenter using calibrated electronics that converted voltage from the potentiometer into angular rotation. The pointer lay in the mid-sagittal plane at the start of every trial.

3.1.2. Stimuli

Stimuli were created using OpenGL graphics libraries and rendered using anti-aliasing and geometric perspective projections from each eye. This ensures delivery of geometrically correct binocular disparities as well as looming information. The stimulus consisted of a wire-frame sphere that was composed of 15 lines of longitude and 15 lines of latitude. The wire-frame construction enabled observers to see both the front and the (unoccluded) back parts of the sphere. The sphere rotated around its centre at 5°/s around its x-axis and 25°/s around its y-axis. The rendered object produced a strong subjective impression of 3-D structure. [1] Perspective calculations for each eye's view of the sphere were made using each observer's inter-pupillary separation. The physical size of the sphere presented to observers was varied: Big spheres had a radius of 2 cm (initially 2.39° at the cyclopean point), Medium spheres had a radius of 1 cm (1.19°) and Small spheres had a radius of 0.4 cm (0.48°). Apart from the target spheres, two references spheres (radius 2 cm) were presented on each trial. The reference spheres were vertically aligned with the starting point of the target; one was centred 10 cm (11.8° from the cyclopean point) below the target and the other 10 cm above. The centre of each reference sphere was in the plane of the screen. On some trials a fixation cross (36×36 arcmin) was presented in the plane of the screen coincident with the initial centre of the target object.

At the start of each trial the target sphere was horizontally and vertically centred on the screen with its centre lying in the plane of the screen. The target moved out of the plane of the screen towards the observer on one of six different trajectory angles with respect to the mid-sagittal plane (2°, 4°, 8°, 16°, 32°, 64°). The elevation of the target did not change during the trajectory (0° with respect to the transverse plane of the head). The magnitude of the Z (parallel to the mid-sagittal plane) or X (perpendicular to the mid-sagittal plane, and horizontal) component of motion for a given trajectory was randomly assigned to be within the range of 9–15 cm. The object's speed was also randomly assigned to be 9, 12 or 15 cm/s. Thus trials lasted different lengths of time (range 450–1617 ms) and there was no simple mapping between the trial duration or the size of the X or Z motion component and the trajectory angle.

We studied one experimental condition in which retinal size and disparity information were deliberately put in conflict. In this condition the angular size at the cyclopean point was held constant at 2.39°. This corresponds to a physical sphere that shrinks as it approaches the observer.

[1] In pilot experiments we investigated the effects of using cylinders rather than spheres, and also the effects of the local rotations of the object on the perception of trajectory. We observed no effects of either manipulation. Spheres were chosen due to the past literature on ball catching, and local object rotations were chosen as the subjective impression of 3-D structure was enhanced.

3.1.3. Validity of the response measure

Most psychophysical procedures measure the *precision* of perception through simple forced choice decisions (e.g. see McKee & Watamaniuk, 1994 for a review of the measurement of human motion perception). A test stimulus is typically judged with respect to a reference stimulus. Measuring the *accuracy* (or bias) of perception is less straightforward. Absolute judgments are required that typically involve other systems such as memory or motor control. The use of absolute measures also entails knowing the validity of the response measure. In the case of trajectory perception, how can we be sure that a chosen measure accurately reflects the observer's perception? Using any particular task there could be large biases in an observer's performance; however, these biases might reflect bias in other aspects of performing the task, not bias in the perception (e.g. bias in motor output). In essence this is a philosophical problem of perception: if we ask an observer what they see can we believe what they say? Whilst we cannot know an observer's perception, we can take two approaches to addressing this problem. First, we can use a range of different tasks and look for correspondence between tasks. Second, we can manipulate stimulus parameters so that we find a situation in which observers can perform a given task with minimum response bias.

To validate their measures of the perception of angular trajectories Harris and Dean (2003) employed four different tasks. Two of their tasks (drawing task, pointer task) involved observers making an explicit judgment of the motion trajectory. In the drawing task, observers drew the perceived motion trajectory (as if from a top–down view) using pencil and ruler, on paper marked with a semi-circle that represented all possible motion directions. In the pointer task, observers used a wooden beam pivoted at the centre of a block of wood that lay on the desk to indicate the motion trajectory from a top–down view. Observers were told to use the pivot point of the beam to represent the starting point of the motion trajectory. The other two tasks employed by Harris and Dean (passing distance interception task, verbal task) involved an implicit calculation of angular trajectory. The passing distance interception task involved observers placing their finger on a beam (mounted across their foreheads, parallel with the plane of the face) to indicate the point at which a viewed trajectory would pass them. The verbal estimation task involved observers indicating whether the object moving towards them would hit their heads. The first two tasks clearly require a complex co-ordinate transformation for their completion (distance, elevation and scale were different between the visual stimulus and the response action), the second two do not. Harris and Dean found observers to be inaccurate for all tasks, with biases always in the same direction (angles were reported as wider than physically specified) and argued that this demonstrated that at least part of the bias must be due to perceptual error.

In this study we adopted a pointer task similar to the one described above. To reduce the complexity of the co-ordinate transformation, we placed the pivot of the pointer at the virtual starting point of the object's motion trajectory. After viewing a trajectory, observers used an unseen hand to move the pointer to indicate their perception of the movement they had just seen. The pointer's axis of rotation was the same as the axis in which object movements occurred, although distance of the hand from the axis did not necessarily match the object's displacement (object displacement was varied randomly—see stimulus description). We used open-loop pointing (observers could not see the hand whilst pointing) to prevent observers simply matching pointer position and final object position. However, to keep the visuo-motor system calibrated, observers were able view their hands and the pointer in its chosen location, after every trial.

To test whether this modified version of the pointer task produced similar results to those obtained with the pointing task used by Harris and Dean, we ran control experiments on four naïve subjects. These subjects made trajectory judgments using a pointer located at the starting point of the motion trajectory, or one located in front of them on the desk. We obtained similar results in both configurations.

To further assess the validity of our response measure, we ran two control experiments. In the first, observers were asked to use the pointer to indicate their perception of the motion trajectory of targets moving in the plane of the screen. The range of angular trajectories was the same as those used for the main experiment, however the objects moved with respect to a horizontal line across the screen (0° elevation), rather than with respect to the mid-sagittal plane. We found that observers showed very little bias when performing the pointer task with the stimulus in this configuration. In the second, we used the verbal estimation task described by Harris and Dean (2003): observers were required to indicate whether an object moving along the presented trajectory would hit their head. The advantage of this task is that it requires no unusual coordinate transformations and the decision can be made on-line, reducing the need for memory. The disadvantage of this task is that only one effective data point is collected for each observer (the point at which the object is perceived as hitting the head on 50% of trials). Results from this experiment were consistent with those reported below: trajectories were overestimated and overestimation was greatest for small objects. These data cross-validate those presented below.

3.1.4. Procedure

Observers sat in a totally dark laboratory and viewed a fixation point whose virtual position was directly in front of them. They initiated a trial by pressing a key with their non-dominant hand, and were presented with a motion trajectory. As soon as possible after the object had disappeared they used their dominant hand to rotate the pointer out of the default position (directly towards them) to indicate their perception of the angle of the trajectory they had just viewed. The experimenter recorded the angle indicated by the pointing devices' electronic display and reset the pointer, at which point the observer could initiate a new trial. A 60 W desk lamp was illuminated after every trial that allowed observers to view their hand, to reduce retinal dark adaptation and to promote continued visuo-motor calibration. Observers were not given any instructions regarding the range of angular trajectories presented to them. They were able to make settings in the range ±90° from the default position.

Observers initially performed a practice block of 10 trials to ensure they grasped the task's requirements. An experimental run consisted of 60 trials (6× angular trajectories, 5× repetitions, 2× experimental conditions). Objects moving on a given trajectory angle were randomly assigned to move leftwards or rightwards from the starting position. Two sets of sphere sizes were randomly interleaved on an experimental run (large [radius = 2 cm] and small [radius = 0.4 cm] spheres, or medium [radius = 1 cm] and conflict spheres [constant angular size at cyclopean point = 2.39°]).

During an experimental run observers were instructed either to pursue the target (no fixation cross presented during the trajectory) or keep their eyes at a fixation cross. We did not measure eye movements, however when subsequently questioned, subjects did not report difficulties in keeping their eyes at the fixation cross when required. Whilst subjective reports of eye position are not always reliable (especially from relatively unpractised observers), the changes in the objects version or vergence were often quite large (making differences between fixation positions obvious). Author AEW (an experienced observer with previous knowledge about the relationship between his subjective reports of eye posture and objective measurements) did not find fixating the fixation cross in the presence of object motion demanding.

Each observer performed four experimental runs; with rest periods between runs, the duration of each observer's participation was about 1 h. The order of experimental runs was randomised across subjects.

3.1.5. Observers

All 11 observers were naïve and paid for their participation. They were recruited from a subject pool in Tübingen, so most had previous experience of psychophysical experiments. Subjects had normal or corrected to normal visual acuities, and good stereopsis (as assessed by the Stereo Fly testing package). Age ranged from 16 to 28 years (mean = 22.5 years).

3.2. Results

To characterise the relationship between the reported angular movement of the approaching object and the one specified by the computer software, we calculated the mean reported angle for each presented trajectory for each subject. Fig. 2 shows an example data set from one observer where the reported angle (α) is plotted as a function of the presented angle (β). The separate plots represent different experimental conditions, and the dashed diagonal line ($x = y$) represents veridical performance. From Fig. 2 it is apparent that performance does not lie along the dashed line. The observer is biased, predominantly reporting that the trajectory angle is larger than that presented ($\alpha > \beta$: data points fall wide of the solid line). However, this is not always the case—for the widest angles (greater than 32°) bias is negative, and the observer underestimates the angle of approach. Although the observer is biased, he is quite precise in his reports: the standard error of the mean is small, and the error bars representing this mostly lie within the plotted symbols.

To characterise the data obtained from all the observers we calculated the mean, between-subjects,

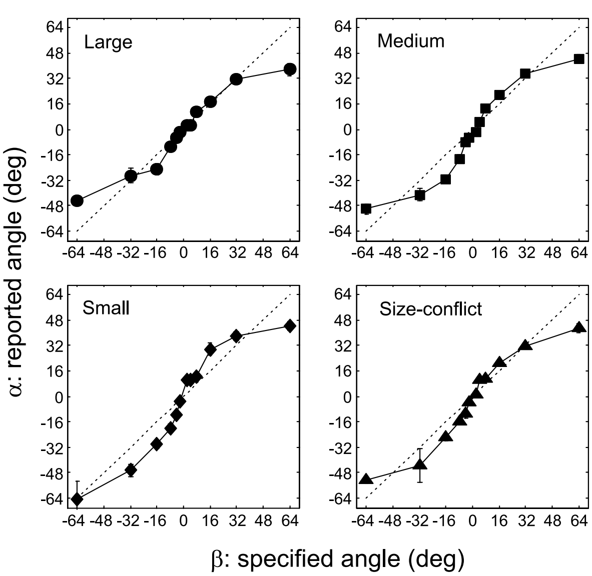

Fig. 2. Reported angle as a function of presented angle for one observer. The separate plots represent the four types of stimuli presented to the observer. The dashed diagonal line on each plot represents veridical performance where reported angle = presented angle. Error bars (many within the symbols) represent the standard error of the mean. The data shown in this figure are pooled across conditions in which the eyes tracked the moving sphere and those in which observers fixated a central marker.

reported angle (α), and plotted this as a function of the presented angle (β). For the purposes of the analysis we ignored the sign of the trajectory angle and calculated mean values based on the unsigned values of α and β. A repeated-measures general linear model (with sign of movement, sphere size and eye movements as factors) conducted on a log transform (used because the untransformed data showed significant deviations from a normal distribution) of the absolute value of α suggested that this was justified as there were no significant differences between α when the direction of motion was leftwards or rightwards ($F_{1,10} = 3.09$, $p = 0.109$). The data showing the relationship between unsigned α and unsigned β across all subjects are presented in Fig. 3.

The data presented in Fig. 3(A)–(C) follow a similar pattern to those from the individual subject whose data were shown in Fig. 2 (although note that only half the function is shown as the data in Fig. 3 have been collapsed onto positive axes). Bias appears to vary as a function of the angle of approach, with maximal positive bias around 16°. For angles greater than 30°–40° observers *underestimate* the approaching angle of an object (data fall below the dashed line). We now address different aspects of the data relating to the experimental questions under consideration.

3.2.1. Do looming cues improve accuracy?

The data presented in Fig. 3 show an ordering effect suggesting differences in the degree of bias in the observers' reports that depends on the size of sphere presented to them. Bias appears smallest for the large (radius = 2 cm) spheres. The reported angle was significantly affected by the size of the presented sphere ($F_{3,30} = 9.26, p < 0.001$); these differences were limited to differences between the big sphere and the three other sphere types (contrast analyses, $p < 0.001$). As the larger spheres are likely to provide more reliable looming information, this is consistent with looming cues improving accuracy. [2]

To determine whether there was a difference between the observers' performance and veridical performance, we performed a linear regression of $\log \beta$ on $\log \alpha$ for each sphere size (see Fig. 3D for the between-subjects mean data). The regression of $\log \beta$ on $\log \alpha$ was highly

[2] It should be noted there was potentially an additional cue to the distance moved in the large sphere condition as the simulated physical size of the target sphere and the references marks (spheres) was the same (i.e. same retinal size at the start of the trial). Observers could potentially have made a judgment about the relative sizes of the objects to estimate the distance moved in depth. To determine whether this could explain the smaller bias found for large spheres (Fig. 3) we ran a control experiment in which subjects judged the trajectories of large and small spheres in the presence of elongated cylinders as reference markers. The results were in accord with those in Fig. 3, suggesting that a relative size cue is not critical in accounting for smaller bias for large spheres.

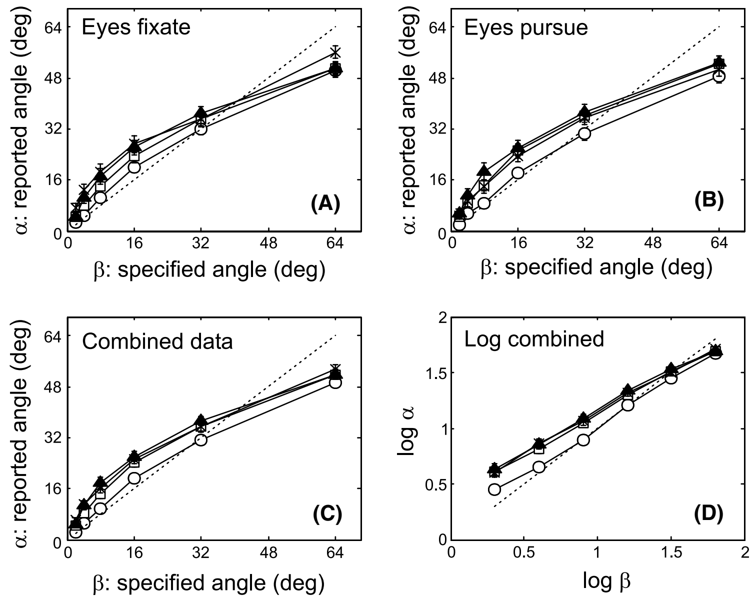

Fig. 3. The relationship between presented and reported angle for four ball sizes with and without the eyes moving. (A) Data obtained when the observers fixated a fixation cross. The data for the four different sphere types are presented: circular markers—large spheres; square markers—medium spheres; cross markers—small spheres; triangle markers—conflict spheres. Error bars represent the standard error of the between-subjects mean. (B) Data obtained when observers pursued the moving object. Symbols and error bars as in (A). (C) Data from pursuit and fixation conditions combined. Symbols and error bars as in (A). (D) Mean log (reported angle) for the four sphere sizes (data combined across fixation and pursuit conditions). Symbols as in (A). Error bars represent the between-subjects mean of log(α).

significant ($p < 0.001$) in all cases. 95% confidence intervals for the slope estimates for each sphere suggested that performance was not veridical. Specifically, slope estimates were less than unity (veridical performance): large sphere coefficient = 0.84 (95% CI: 0.80, 0.89); medium sphere coefficient = 0.74 (95% CI: 0.69, 0.78); small sphere coefficient = 0.73 (95% CI: 0.68,0.78); conflict sphere coefficient = 0.72 (95% CI: 0.67,0.77). Observers appear to produce non-veridical reports of trajectory in all cases when looming and/or disparity information is available. This analysis also allowed us to compare the slope estimates for the different sphere sizes. Non-overlapping confidence intervals for the large sphere slope estimate compared with the other sphere types indicated that performance was closer to veridical in the large sphere case, whereas there were no differences between the other sphere types.

3.2.2. Is tracking the moving object important?

In Fig. 4 we present data on the effects of moving the eyes on observers' judgments of trajectory angle. Each plot shows the data for a different sphere size, and the separate series show data obtained when observers tracked the object or fixated a point in the scene. All data, apart from those collected with the small spheres, shows a very close overlap between data collected when they eyes were free to follow the object and when observers were required to fixate. The repeated-measures GLM suggested that the effects of eye movement were not significant ($F_{1,10} = 2.97$, $p = 0.115$). Inspecting the figure might suggest that observer performance was closer to veridical when they tracked the small spheres, however, no significant interaction between sphere size and eye movement was observed ($F_{3,30} = 2.52$, $p = 0.077$).

3.3. Discussion

In this experiment we used computer-rendered textured spheres to study the ability of human observers to judge the angular trajectory of an approaching object. This was done to confirm and extend a study by Harris and Dean (2003) in which the visual information available to observers was minimal. We manipulated the size of the approaching object to investigate the utility of looming information, and also studied the influence of eye movements on judgments of trajectory. Our main findings are (1) observers overestimate the angle of approach of an object when the object moves at a small angle (<30°) with respect to the mid-sagittal plane; (2) judgments are less biased when larger objects are used (>1 cm in radius); (3) eye movements appear to make little difference to observers' judgments.

The results we present here are consistent with those of Harris and Dean (2003). They measured trajectory angles of less than 45°, using small stimuli (8.3 min) where no looming information was available, and found overestimates of trajectory angle. The fact that similar biases were observed in both studies suggests that the effects observed when looming was not available cannot

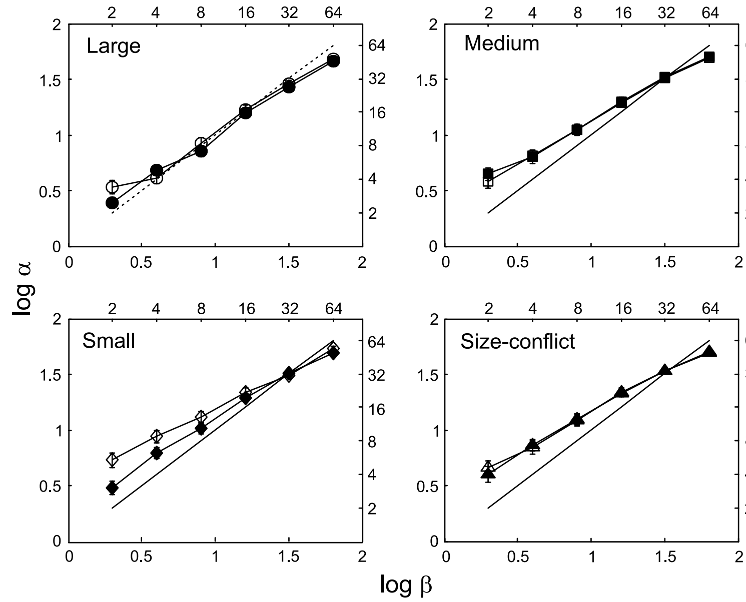

Fig. 4. Effects of eye movement on trajectory judgments. Filled symbols show moving eyes and open static eyes. Separate plots represent the different sphere sizes. Error bars represent the standard error of the mean of log(α).

be explained by the argument that use of a single, isolated cue is the main cause of bias.

There are some minor differences in our data. In particular, Harris and Dean did not observe that bias changed dramatically as a function of the approach angle. Indeed, for their larger angles (45°) observer responses appeared to saturate at 90° (they perceived larger angles as frontoparallel). In contrast, we report that bias becomes less positive and eventually negative as the angle of approach is increased. But note that all the stimuli presented here contained both looming and binocular disparity information. Could looming be responsible for the different pattern of data? There is a suggestion from the data that the point at which the sign of the bias changed varied according to the size of the approaching object. For large objects the intersection of the veridical performance line and the function relating α and β appear to be at a smaller angle (ca. 30°) than for the smaller or size-conflict spheres (ca. 38°–44°). It is possible that for the very small sizes used by Harris and Dean (8.3 arcmin), a change of sign might occur at even larger angles, or be absent due to simple response saturation.

4. Experiment 2: Real-world object movement

In this experiment we investigated the perception of angular trajectory when a real object moves in depth. Our aim was to test whether the cue conflicts arising due to stereoscopic presentation of computer generated images caused biases in observers' judgments of trajectory. Stereoscopic images displayed via a mirror stereoscope or sequential-presentation shutter goggles are never identical to those imaged on the retina when an observer views a real object. Computer generated stereoscopic images can be subject to geometric distortions (for example, pin cushioning, where what should be a straight line is bowed outward at the edges of the display screen). Perhaps more importantly, the accommodative demand specified by the stimulus is that of the plane of the computer monitor, rather than the depth of the object being presented. Also, unless the observer's head is completely fixed, small movements of the head will produce motion parallax of the stimulus that is consistent with the object being in the plane of the screen rather than moving towards the observer. Motion parallax information can provide a powerful cue to the 3-D structure of objects (e.g. Howard & Rogers, 2002; Rogers & Graham, 1979), and some recent work suggests that the blur cue (which drives ocular accommodation) can have an impact on depth perception (Watt, Akeley, & Banks, 2003). Other recent results suggest that the mode of stereoscopic presentation can affect thresholds for motion-in-depth discrimination (Tuck, Welchman, & Harris, 2002). It is therefore important to establish whether the bias reported in Experiment 1, and in Harris and Dean (2003), could be caused by impoverished estimates of depth caused by the mode of presentation.

4.1. Methods

4.1.1. Apparatus

Real world stimuli were presented on a Werner GmbH PA-Control double axis, linear track. The equipment uses a pair of linked stepper motors that can be programmed to move a platform independently along a pair of perpendicular axes. The platform can be programmed to move along each axis for a specific distance in regular increments (minimum 0.1 cm) at a range of different speeds. This allowed us to present 3-D trajectories with specific X and Z components of motion. The linear track was bolted to a solid wooden lab-bench to reduce vibration and keep unwanted movement to a minimum.

Visual stimulation was provided by light emitting diodes (LEDs), 0.3 cm in diameter, that were illuminated by current supplied from a regulated DC power supply. The LEDs were red in colour and measured luminance was 0.1 cd/m^2. When not illuminated, an LED subtended a visual angle of 7.9 arcmin at the viewing distance of 130 cm. One LED was mounted on top of a post attached to the platform of the track. A second LED was mounted 3.5 cm directly above the LED that was mounted on the platform. This second LED was used as a fixation mark for the subjects, and it provided a reference against which the observer could judge the direction of motion of the LED mounted on the track's movable platform. The LEDs were switched on and off automatically when required by wiring them to the stepper motor controllers. To cut down any reflections of the light from the LEDs on the surfaces of the apparatus, the posts holding the LEDs and the lab-bench itself were covered in matt black fabric and black tape. A black board barrier was positioned directly in front of the observer's chin to ensure that only the LEDs were visible.

In order to keep the observer's head in the correct position throughout the experiment, a chin rest was mounted on the lab-bench. The height of the chin rest was varied to ensure that the reference LED was positioned at eye height for each observer. The chin rest was located along the Z-axis of the track's motion, which approximately corresponded to the mid-sagittal plane of the observers.

Measures of the perceived motion trajectory were collected using a manual pointing device that consisted of a pointer pivoted at the centre of a square, wooden mount (30×30 cm) (much like a clock face with one hand). The default position for the pointer was towards the observer through the mid-sagittal plane (i.e. 6 o' clock). The mount did not have any markings visible to the observers apart from one indicating the default position of the pointer. The pointing device lay flat on the lab-bench 29 cm directly in front of the observer. Angular rotations of the pointer were measured by an experimenter using a scale invisible to the observer.

4.1.2. Stimuli

The visual stimuli consisted of the two illuminated LEDs described above. One of the LEDs had a fixed position in space (130 cm in front of the observer level with the eyes) and was used by observers as a fixation marker. The other LED was the target LED positioned 3.5 cm (1.5° at the starting location) beneath the fixation LED. The target LED was small enough for us to be confident that it did not contain a useful size-change cue when moving in depth. The target LED subtended a visual angle of 7.9 arcmin at a viewing distance of 130 cm (more than three times smaller than the smallest sphere used in Experiment 1). If the target were to move forwards by 13.2 cm it would subtend a visual angle of 8.77 arcmin. The change in size of the object during the movement would be 0.87 arcmin (52 arcsec). In order to provide a strong looming cue, evidence suggests that a larger object should be used (e.g. Regan & Beverley, 1979; Rushton & Wann, 1999). Using a looming target oscillating at 3 Hz, Regan and Beverley (1979) found that the change in size must be at least 0.7 arcmin to produce the sensation of motion in depth. Thus the looming cue presented in the following experiment is very close to threshold.

At the start of each trial the target appeared for 1 s directly below the fixation LED. Motion towards the observer at a constant speed was then presented for 3 s. Four different trajectory angles with respect to the mid-sagittal plane were examined: 5°, 10°, 15° and 20°. The elevation of the target did not change during the motion trajectory. The trajectory angle of the LED was manipulated by varying the X-component of motion between 1.2 and 4.8 cm; the Z-component of motion was constant at 13.2 cm. The speed of motion on the different trajectories was varied so that the duration of each trial was 3 s. These parameters were chosen to be similar to those used by Harris and Dean (2003).

4.1.3. Procedure

Observers sat in a totally dark laboratory and were asked to fixate either the stationary reference LED or the moving target LED. They were presented with a trajectory after which the experimenter illuminated a 60 W desk lamp and observers rotated the pointer out of the default position (directly towards them) to indicate their perception of the approaching LED. Lighting was arranged to be below the desk holding the track, so that observers could see the pointer, but not the unlit target LED. Observers were required to move the pointer to indicate the motion direction, as if viewed from above. No instructions regarding the range of angular trajectories presented to them were given, and observers were able to make settings in the range ±90° from the default position. The experimenter extinguished the desk lamp before the presentation of the next trajectory. Motion trajectories always started from the same position. The re-positioning of the target LED to the starting location was not visible to the observer as the LED was not illuminated.

An experimental run consisted of 36 trials (9× angular trajectories: 0, ±5, ±10, ±15, ±20; 4× repetitions). Each observer performed two runs; this took around 40 min. On one of the experimental runs observers were required to pursue the moving LED with their eyes, rather than fixating the stationary LED. The order of experimental runs was randomized across subjects. Eye movements were not recorded.

4.1.4. Observers

All nine observers were naïve, and recruited from staff and students in the School of Biology (Psychology), Newcastle. Observers had normal, or corrected to normal vision. Age ranged from 25 to 52 years.

4.2. Results

The form of the results is similar to those presented for Experiment 1. We calculated the mean reported angle between observers for all the trajectory angles presented. In Fig. 5A we plot the between-subjects means for the reported angle (α) as a function of the physically presented angle (β). The diagonal ($x = y$) line represents veridical performance. Filled squares show data when observers were asked to pursue the moving target, open circles show data when observers were asked to fixate the stationary reference. It can be seen from this graph that observers report a larger trajectory angle than was presented (i.e. $\alpha > \beta$) in all cases except when motion is directly towards them ($\beta = 0 = \alpha$). As the function relating α and β is approximately linear, we performed a regression of β on α to determine whether the slope of the function was significantly different from unity (veridical performance). The regression was highly significant ($F_{1,574} = 1023.28$, $p < 0.001$), and estimated slope of the function differed from unity (slope = 1.46; 95% CI: 1.37, 1.55), confirming that performance was not veridical.

The pooled, log-transformed data are shown in Fig. 5B. For all the presented trajectories, the data lie above the veridical performance line, indicating that observers overestimate the trajectory angle of the approaching target. To investigate the influence on eye movements on reports of trajectory direction (log α) we used a repeated-measures general linear model with direction of motion and eye movement as factors (logs were used to satisfy normality requirements of the procedure). Neither the direction of motion ($F_{1,8} < 1$, $p = 0.44$) nor the effect of eye movement ($F_{1,8} = 1.27$, $p = 0.29$) was significant.

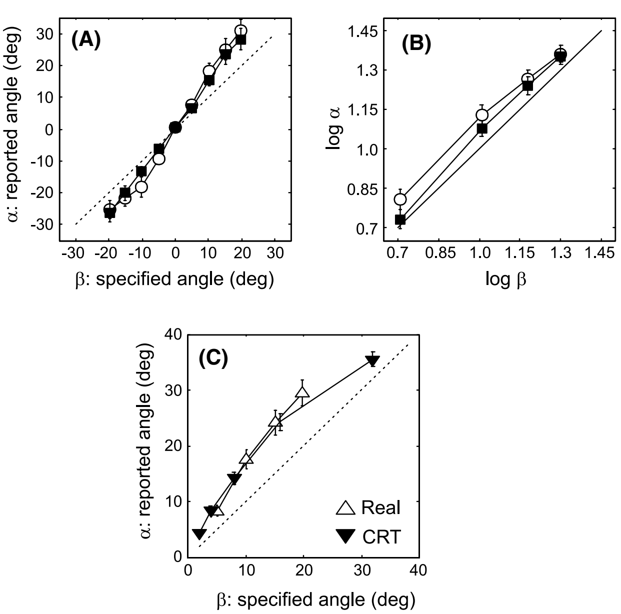

Fig. 5. Results from the experiment using real-world motion. Filled squares show results obtained when the observer was instructed to pursue the target, open circles data collected when observers fixated a reference mark. (A) Mean between-subjects reported angle (α) as a function of presented angle (β). Error-bars show the standard error of the mean. (B) Mean between-subjects logarithm of the magnitude of the reported angle (α) as a function of the logarithm of β. Error bars show the standard error of the mean. (C) A comparison between data obtained for real-world motion and motion presented on a computer screen (CRT). These data are between-subjects mean data, combining moving and static conditions for the data collected in Experiment 2 (open triangles) and the small spheres in Experiment 1 (filled triangles). Error bars ± SEM.

4.3. Discussion

There are potentially a number of factors that complicate the use of computer screens to generate stimuli that portray 3-D structure (e.g. inappropriate parallax, accommodative conflicts), and which could cause observers to produce performance that is not veridical. To determine whether bias in judgments of motion trajectories could be explained by these factors we performed an experiment where we presented real 3-D movement of small points of light. We observed significant bias in the observers' responses, just as we did when using stereoscopic images presented via a computer display. To compare the results of Experiments 1 and 2 we present data plotted on the same graph in Fig. 5C. There is a remarkable similarity between the results of the two experiments, despite the fact that data were collected on different set-ups with different observers and different ranges of presented angles. It therefore appears unlikely that the mode of presentation can explain the mis-estimation of trajectories.

In Experiment 2 we again investigated the influence of eye movements on trajectory judgments in this experiment as changes in accommodation (absent in Experiment 1) are known to be important for the programming of the vergence eye movements (e.g. Judge, 1991; Judge & Miles, 1985) required when observers are asked to pursue the moving target. Similar to the previous experiment, we found no significant effect of eye movements. However, like the results from Experiment

1 for the smallest spheres, inspecting Fig. 5B suggests a slight improvement in accuracy when observers pursued the target. A possible explanation for this effect relates to the importance of keeping the object foveated. For larger objects, size changes could be registered sufficiently well using peripheral detection mechanisms; however for smaller sizes foveal viewing might be critical. If detecting small changes in size (i.e. looming) or small differences in the retinal extent in the two eyes (e.g. HSRs) is important, then keeping the object foveated would result in greater sensitivity to these cues.

5. General discussion

In this paper we have examined human observer's judgments of motion-in-depth trajectories in the presence of both looming and binocular disparity information. We found that the presence of both these cues, and the absence of conflicting accommodation and motion parallax cues, does not lead to veridical performance. The data we obtained are generally consistent with those presented by Harris and Dean (2003): observers show a positive bias, overestimating the angle of approach when movement is near the mid-sagittal plane. We now discuss our findings in detail.

5.1. Why is size of the object important?

In the first experiment we presented evidence that observers' reports of trajectory angle are closer to veridical when larger objects are used (observers showed less bias for large spheres than for the medium or small spheres). Also, when observers were presented with a sphere that was initially the same size as the large sphere, but whose size at the cyclopean point remained constant (corresponding to a ball in the real world that shrinks as it approaches the observer), observers showed a significantly larger bias. This evidence suggests that changing size (or looming) cues are used to help determine the trajectory, and help reduce bias. The effects of object size are informative because, for the same distance moved in the world, the changing-size information for a large object is likely to be more reliable (the effects of noise will be proportionately less) than for a smaller object. It appears that making the looming cue more reliable leads to improved performance. Why might this be so?

In our mathematical analysis at the beginning of the paper we showed that trajectory angle can, in principle, be calculated from looming cues alone on the basis of uncalibrated retinal information. This contrasts with the use of binocular information for which an estimate of the viewing distance and the interocular separation are required. If the visual system does not have access to these, or estimates them incorrectly, trajectory would also be specified incorrectly. A number of previous studies (e.g. Foley, 1980; Johnston, 1991) have suggested that observers frequently use an incorrect estimate of viewing distance to interpret disparity information (often referred to as incorrect disparity scaling). Thus, we could speculate that observers in this study obtained a biased estimator of angular trajectory from binocular information (e.g. HSRs), whereas they obtained an unbiased estimator from looming information. If these two estimators were combined (Landy, Maloney, Johnston, & Young, 1995) using maximum likelihood estimation (Ernst & Banks, 2002; Yuille & Bülthoff, 1996), then the weight given to the looming estimator should increase as the reliability of that estimator increases (i.e. as the stimulus size increases). With a more reliable looming estimator, the bias of the combined estimator could be reduced. It is of course possible that the measurement of binocular information (such as HSRs) also becomes more reliable with larger objects. This argument still works as long as the reliability of the looming estimator increases more than the reliability of the estimator derived from binocular cues (i.e. the relative weights of the two cues change).

Vertical disparities have been suggested to provide a retinal measure of the viewing distance needed to scale horizontal disparities (e.g. Longuet-Higgins, 1982; Rogers & Bradshaw, 1993). Increasing the size of the object could have increased the reliability of the estimate of viewing distance derived from vertical disparities in the stimulus, potentially explaining smaller bias for the large spheres. However, we think this unlikely as our stimulus was never more than 7° in size, and evidence from the literature suggests that the stimulus extent should be greater than 10° for vertical disparities to be useful (Bradshaw, Glennerster, & Rogers, 1996). Also, Brenner, Smeets, and Landy (2001) provided evidence that gradients of vertical disparity, rather than the disparities themselves, are useful for distance scaling. Our stimulus would have provided poor information about disparity gradients.

Whatever the cause of the advantage of using a larger object, note that the advantage is small. Performance did not become veridical, even with the largest object used.

5.2. Why does bias change as a function of trajectory angle?

Under all conditions in Experiment 1, we observed that the degree of bias changed as a function of the trajectory angle (e.g. Fig. 3). Although, in principle, there are several reasons why the precision of observers' estimates might change as a function of the presented trajectory angle, we do not have a satisfactory explanation for why bias should vary as a function of angle. For example, increased precision could result from more reliable information being obtained from horizontal size ratios. The magnitude of the HSR depends on both the distance and the eccentricity of the stimulus (Gillam & Lawergren, 1983). Under the conditions studied here, a motion-in-depth trajectory that makes a large angle (e.g. 64°) will start with $HSR = 1$ (the object always started from the mid-sagittal plane) and end with a larger ratio ($HSR = 1.04$). However, a trajectory on a small angle (e.g. 4°) will result in a very small change of HSR (HSR at the end of the trajectory would be 1.005). If observers were using HSRs then more reliable estimates of trajectory might be expected from larger changes in HSR than smaller ones, leading to increased precision; but bias would not be affected. We found no evidence for increases in precision with wider angles (standard errors of the mean reported trajectory did not vary systematically with presented trajectory), but we did find that bias changed as a function of trajectory. Clearly, this runs counter to the expectations of calculations based on HSR, as was put forward in the mathematical analysis presented earlier.

5.3. Does the visual system actually calculate angular trajectories?

Humans are generally proficient at intercepting or avoiding approaching obstacles; however this skill does not necessitate the reconstruction of the objects' precise spatio-temporal trajectory. For instance, an observer could determine whether or not they will be hit by a ball thrown towards them by monitoring the changing position of the ball over time with respect to the thrower: if there is little change in the visual direction of the ball and the visual direction of the (static) thrower—then the ball is likely to be on a collision course with them. This is a real-world example of a relative method for obstacle avoidance that avoids the need to make absolute judgements about the trajectory of a moving object. Another example comes from work on catching real balls. Peper, Bootsma, Mestre, and Bakker (1994) showed that predictive information about the future position of a ball is not necessary for ball catching. Instead, they showed how observers could use continuous action related information. The experiments presented in this manuscript reinforce this suggestion that relative and/or continuous judgments may be more useful to observers as, under many circumstances, such judgements avoid the problem of measurement bias.

The evidence for measurement bias presented in this paper is necessarily indirect. Although we know that observers responses are biased, we cannot know for certain whether an observer's perception of an angular trajectory was also biased, or whether only their reports of their perception were biased. We have, however, observed consistently biased reports under quite a range of viewing conditions, with a variety of visual cues to depth, distance and motion, and under different task constraints (see also Harris & Dean, 2003). This suggests to us that at least part of the bias observed reflects a genuine perceptual bias.

In summary, human observers are highly sensitive to differences between the angular trajectories of two sequentially-presented motion-in-depth trajectories (Beverley & Regan, 1975; Regan & Kaushal, 1994). However they do not necessarily use these cues to calculate the spatio-temporal trajectory of the moving object. In this study, although we presented observers with information that is mathematically sufficient to specify the angular trajectory of the approaching object, observers showed large biases in estimating trajectory. The data from our absolute task leads us to question whether explicitly calculating the angular approach of an object is something that humans routinely do.

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Appendix A

The aim of this section is to express trajectory direction in terms of horizontal size ratio (HSR)—a formulation of the use of binocular information that involves the relative size differences between the projected images of a ball in the two eyes (see discussion of the use of this term in Section 1 of this paper).

A ball of radius s, is located in the medial plane of the head a distance d from the observer (Fig. 6). The ball moves a distance P at an angle β to the visual midline. The distance moved can be decomposed into a component perpendicular to the mid-sagittal plane, X, and a component of motion in the mid-sagittal plane Z. We will express the calculations of angle β in terms of the distance moved (we are agnostic as to whether the visual system would be more likely to use a formulation based on motion or on distance moved). First we derive expressions, in terms of information at the retina, for the X and Z distances moved. We then obtain an estimate for β from those.

First, assume that the visual system is able to measure the visual direction of the centre of the ball with respect to the cyclopean point. This could be done using a

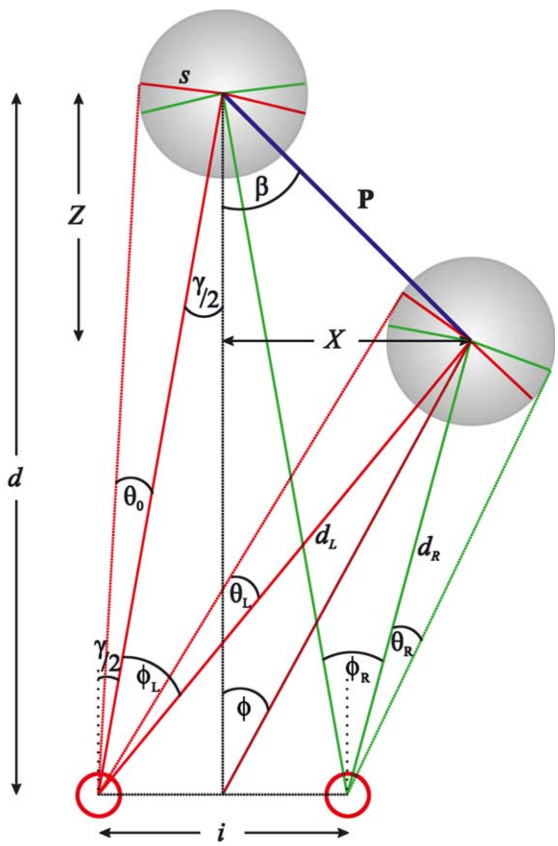

Fig. 6. Quantities used in the derivation of the equations based on changing size and changing relative size information.

common approximation for the visual direction of the centre of the ball (Howard & Rogers, 2002): [3]

$$\phi \approx \frac{\phi_L + \phi_R}{2}$$

that is related to the X and Z distances moved by

$$\tan\phi = \frac{X}{d - Z}, \quad \text{or, rearranged: } Z = d - \frac{X}{\tan\phi} \qquad (A.1)$$

Now, let us define the azimuth of the centre of the object in each eye, angles λ and ρ: $\lambda = \phi_L + \gamma/2$ and $\rho = \phi_R - \gamma/2$. Let us use these angles to derive expressions for the distance of the centre of the ball from each eye:

$$d_L = \frac{X + i/2}{\sin\lambda} \quad \text{and} \quad d_R = \frac{X - i/2}{\sin\rho} \qquad (A.2, A.3)$$

The angular size of the half of the ball in each eye calculated by

$$\sin\theta_L = \frac{s}{d_L} \quad \text{and} \quad \sin\theta_R = \frac{s}{d_R}$$

which by substituting from Eqs. (A.2) and (A.3) gives

$$\sin\theta_L = \frac{s\sin\lambda}{X + i/2} \quad \text{and} \quad \sin\theta_R = \frac{s\sin\rho}{X - i/2} \qquad (A.4, A.5)$$

We will define the horizontal size ratio (HSR) of the object size in each eye as

$$\mathrm{HSR} = \frac{\theta_R}{\theta_L}$$

if $d_L \gg s$ and $d_R \gg s$ then $\sin\theta_R \approx \theta_R$ and $\sin\theta_L \approx \theta_L$, so, substituting from Eqs. (A.4) and (A.5) we obtain

$$\mathrm{HSR} \approx \frac{\sin\rho(X + i/2)}{\sin\lambda(X - i/2)} \qquad (A.6)$$

Eq. (A.6) can be re-arranged to provide an expression for X:

$$X \approx \left(\frac{i}{2}\right)\frac{\mathrm{HSR}\,\sin\lambda + \sin\rho}{\mathrm{HSR}\,\sin\lambda - \sin\rho} \qquad (A.7)$$

Combining Eqs. (A.1) and (A.7) we obtain

$$Z \approx d - \left(\frac{i}{2\tan\phi}\right)\frac{\mathrm{HSR}\,\sin\lambda + \sin\rho}{\mathrm{HSR}\,\sin\lambda - \sin\rho} \qquad (A.8)$$

Having obtained expressions for X and Z we can now derive the trajectory angle:

$$\tan\beta = \frac{X}{Z} \approx \frac{i\tan\phi(\mathrm{HSR}\,\sin\lambda + \sin\rho)}{2d\tan\phi(\mathrm{HSR}\,\sin\lambda - \sin\rho) - i(\mathrm{HSR}\,\sin\lambda + \sin\rho)} \qquad (A.9)$$

This expression holds if objects are not located in the mid-sagittal plane (unlike derivations presented elsewhere [4]). If the centre of the object is located in the mid-sagittal plane then HSR = 1, and it can be shown that Eq. (A.9) simplifies to

$$\tan\beta \approx \frac{i\phi}{d\delta}$$

where $\delta = \phi_R - \phi_L$ (the binocular disparity), as formulated by Harris and Dean (2003). Note also that if θ_R and θ_L are very small, then the HSR tends to 1, resulting in the same expression.

[3] Visual direction is commonly defined as the average of the azimuth (or eye rotation) in each eye. However, defined in this way the cyclopean point (mid-way between the eyes on the interocular axis) does not lie on the interocular axis unless the object is at infinity. Visual direction defined using this formula corresponds to a measurement made from a point lying behind the interocular axis on the mid-sagittal plane. Its distance behind the interocular axis depends on the distance and the (headcentric) azimuth of the presented object (the point is defined as the intersection of the mid-sagittal plane with a Veith-Müller circle passing through the centre of the object and the nodal points in each eye). The approximation, however, is reasonable under most viewing conditions. If the object lies more than 32.5 cm away from the observer then the error will be less than 1% (approximately 0.2°).

[4] Although note no account is take of the small changes in the effective interocular separation with changes of azimuth.

We will now derive an expression for angular trajectory based on changes in mean size. For the binocular use of changing size information it is convenient to consider changing size information with respect to a point half way between the two eyes on the interocular axis (known as the cyclopean centre).

We will assume that the visual system is able to make an estimate of the binocular subtense of the ball at the cyclopean centre by averaging the sizes in the left and right eyes. This provides a reasonable approximation of angular size under most circumstances. [5] Angular size of half the object is given by

$$\theta \approx \frac{\theta_L + \theta_R}{2}$$

The initial size of half the object can calculated as

$$\sin\theta_0 = \frac{s}{d} \tag{A.10}$$

The size once the object has moved distances X and Z depends on the distance to the centre of the object from the cyclopean point: d_C. This distance can be calculated as

$$d_C = \sqrt{X^2 + (d - Z)^2}$$

Thus, a more general form of Eq. (A.10) is

$$\sin\theta = \frac{s}{\sqrt{X^2 + (d - Z)^2}}, \text{that rearranged gives:}$$

$$Z = d - \sqrt{\frac{s^2}{\sin^2\theta} - X^2} \tag{A.11}$$

Eqs. (A.11) and (A.1) can be equated to yield and expression for X

$$X = \frac{s\tan\phi}{\sin\theta\sqrt{1 + \tan^2\phi}} = \frac{s\tan\phi}{\sin\theta\sqrt{\sec^2\phi}}$$

given that $\sec\phi_C = (\cos\phi_C)^{-1}$ and $\tan\phi_C = \sin\phi_C / \cos\phi_C$,

$$X = \frac{s}{\sin\theta}\sin\phi \tag{A.12}$$

Substituting Eq. (A.12) into (A.1) gives

$$Z = \frac{d\sin\theta - s\cos\phi}{\sin\theta} \tag{A.13}$$

Using Eqs. (A.12) and (A.13) we can calculate β:

$$\tan\beta = \frac{X}{Z} = \frac{s\sin\phi}{d\sin\theta - s\cos\phi} \tag{A.14}$$

Using Eq. (A.10) we can substitute for s:

$$\tan\beta = \frac{d\sin\theta_0\sin\phi}{d\sin\theta - d\sin\theta_0\cos\phi} = \frac{\sin\theta_0\sin\phi}{\sin\theta - \sin\theta_0\cos\phi} \tag{A.15}$$

if we assume $d_C \gg s$, the small angle approximation yields [6]

$$\tan\beta \approx \frac{\theta_0\sin\phi}{\theta - \theta_0\cos\phi} \tag{A.16}$$

In Regan and Kaushal's (1994) formulation, they made the further assumption that ϕ is small (i.e. $d \gg X$). Using this approximation $\sin\phi \approx \phi$, $\cos\phi \approx 1$, so Eq. (A.16) simplifies to become equivalent to the one they provided: [7]

$$\tan\beta \approx \frac{\theta_0}{\theta - \theta_0}\phi$$

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[5] Angular size defined using this formula corresponds to a measurement made from a point equidistant between the eyes in front of (large azimuth) or behind (small azimuth) the interocular axis. The distance from the interocular axis depends on both the size and the azimuth of the object. For the case under consideration, provided the object is more than 22 cm from the cyclopean point the error is less than 0.01°. An alternative way for the visual system to calculate trajectory is to perform the calculations monocularly, with respect to the information provided by each eye alone, and then average the estimates of trajectory angle obtained from each eye. Whilst this does not necessitate approximations, we formulate the maths based on calculations from a cyclopean point as it seems more biologically plausible.

[6] For our circumstances, this assumption produces an error of ca. 0.1% or less (depending on angular trajectory).

[7] We do not make this assumption as errors were ca. 11% for the largest trajectory angles studied.

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